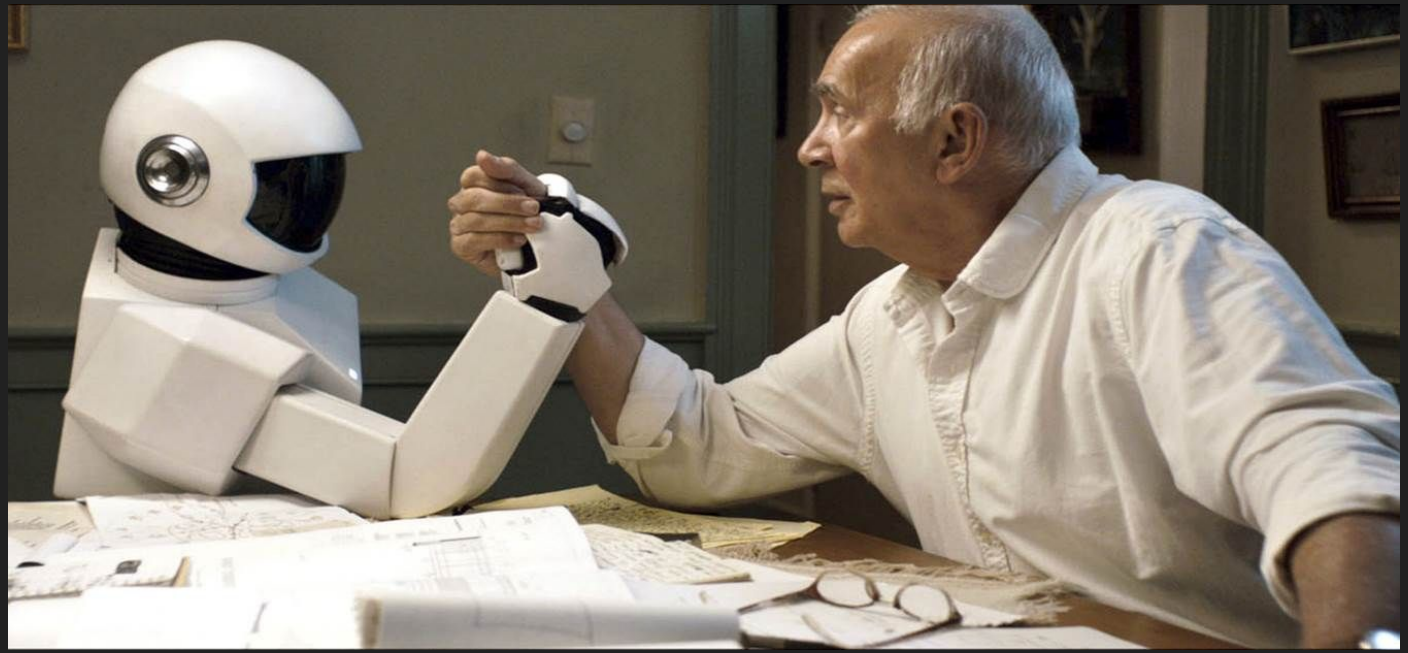
Legibility and Predictability of Robot Motion

So-bots Journal Club

University of Glasgow

What’s the problem?



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Make this easier:

- Making better predictions
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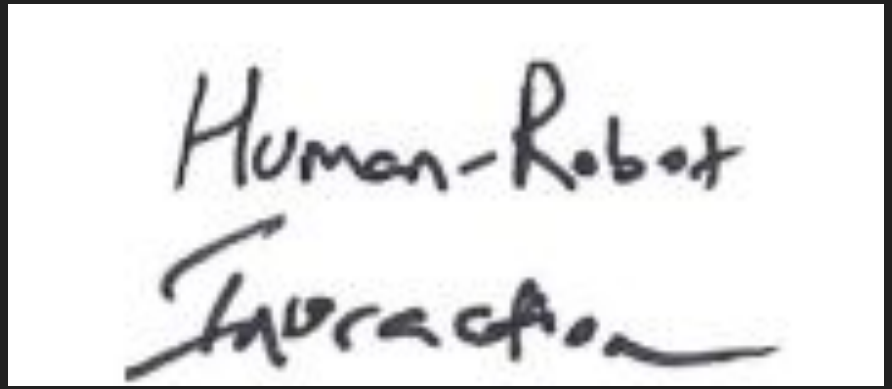
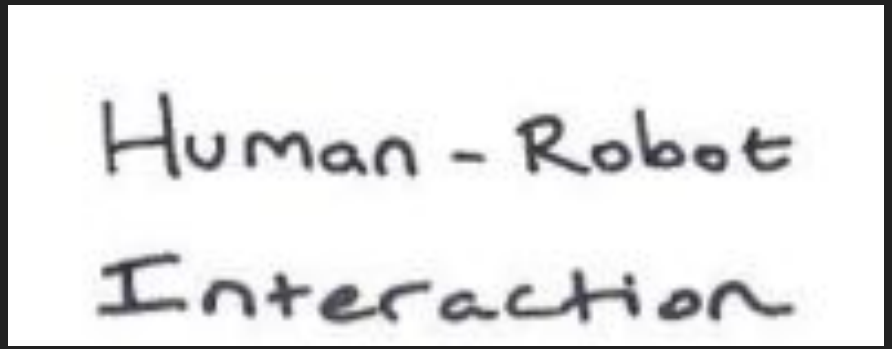
Legible Vs. Predictable - intuitions



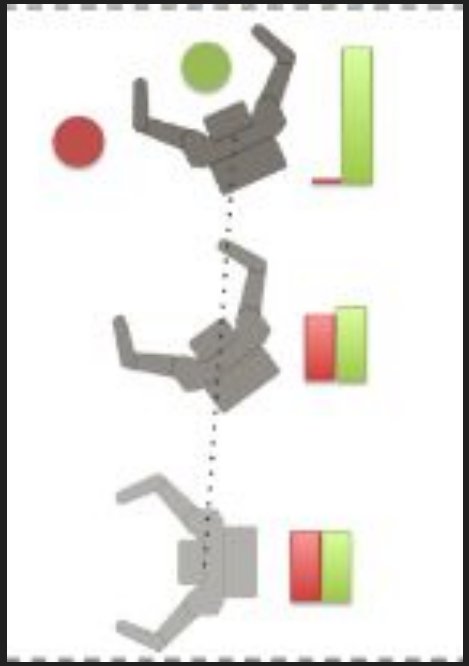
Someone's *predictable* handwriting

Legible Vs. Predictable - intuitions



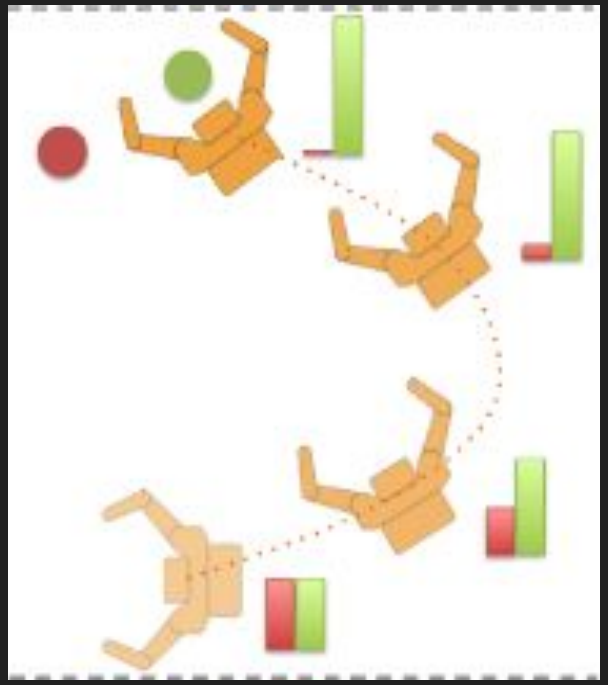
Someone's *legible* handwriting

How does this relate to motion?



Predictable movement.

How does this relate to motion?



Legible movement.

Predictable Movement Formalism

Def.: Predictabel motion is motion that matches what an observer would expect *given a goal.*

$$\mathcal{I}_p : \mathcal{G} \longrightarrow \Xi$$

Set of *all* possible goals

Set of *all* possible trajectories

Predictable Movement Formalism

Def.: Predictabel motion is motion that matches what an observer would expect *given a goal.*

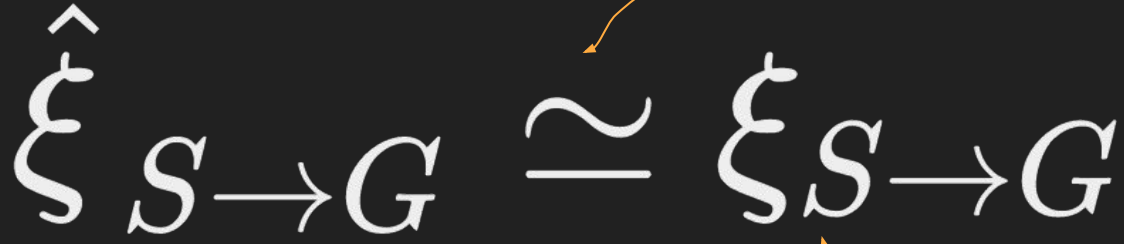
$$\mathcal{I}_p(G) = \hat{\xi}_{S \to G}$$

Inference function on a given goal

Produces an estimated trajectory

Predictable Movement Formalism

A motion is predictable if:



Some distance metric

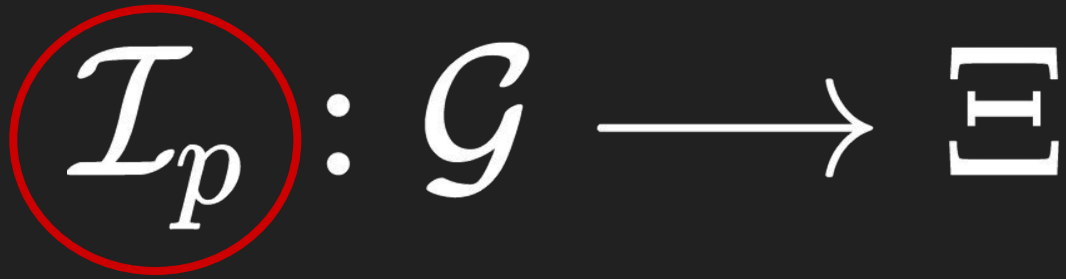
Guessed trajectory given goal

Actual trajectory produced

Predictable Movement Formalism

$$\mathcal{I}_p : \mathcal{G} \longrightarrow \Xi$$

Predictable movement formalism



What is this inference function that the robot has to perform?

Predictable Movement Formalism

$$\mathcal{I}_p$$

Observer's (robot) expectation about the movement the human will produce.

Predictable Movement Formalism

What expectation is reasonable for the robot to make?

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Principle of rational action:
An observer expects a rational agent to act efficiently or justifiably.

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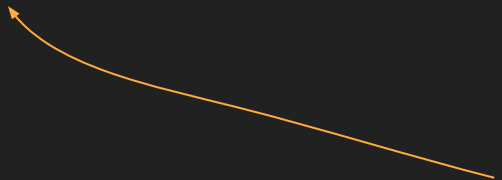


In what sense?

Predictable Movement Formalism

Predictable Movement Formalism

To calculate the cost associated with a trajectory we need a cost function:

Cost function

Set of *all* possible trajectories

Set of positive real numbers

Predictable Movement Formalism

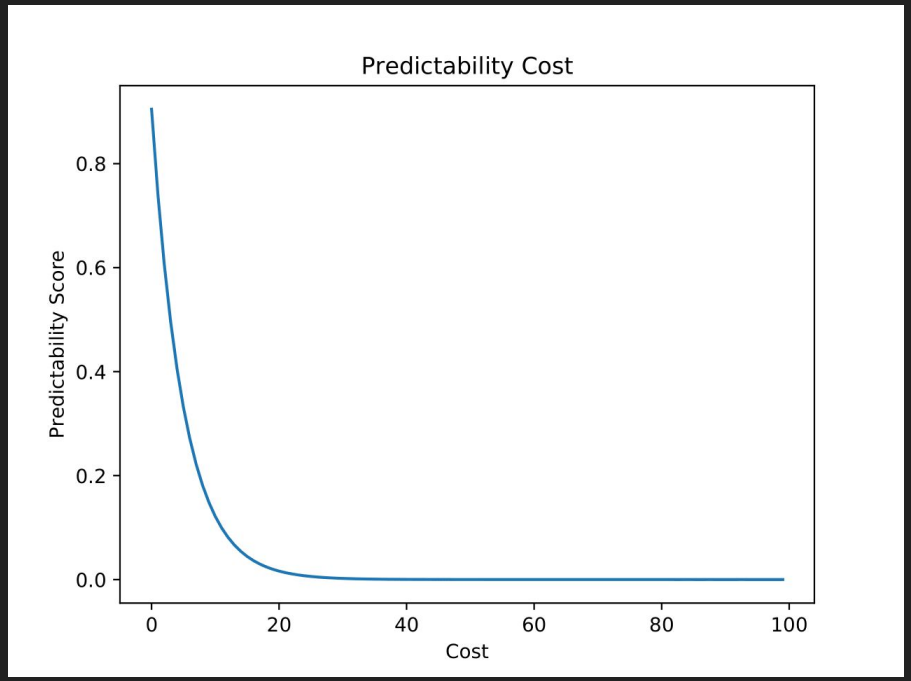
The trajectory that should be predicted by the robot is the one that has the lowest cost with respect to some measure of efficiency:

$$\mathcal{I}_p(G) = \hat{\xi}_{S \to G} = \underset{\xi \in \Xi_{S \to G}}{\arg\min}\, C(\xi)$$

The expected trajectory is the one that produces the lowest cost *out of all the possible trajectories.*

Predictable Movement Formalism

$$predictability(\xi) = exp(-C(\xi))$$



Predictable Movement Formalism

We now have two clear problems to solve:

1. Robot needs to know what cost function the human expects i.e. how does it expect the robot to move?

Predictable Movement Formalism

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1. Robot needs to know what cost function the human expects i.e. how does it expect the robot to move?
2. Need to find the movement that produces the smallest cost.

Legible Movement Formalism

Def.: Legible motion enables an observer to quickly and confidently infer the correct goal from that motion.

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$$\mathcal{I}_L(\xi_{S \to Q}) = \hat{G}$$

"Snippet" of a trajectory ($Q \leq G$)

Estimated goal

Legible Movement Formalism

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Take away: *the shorter the "snippet" of a given trajectory that leads to a correct guess of the goal the better the trajectory.*

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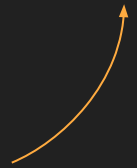

Bring the most *telling* information about the movement's goal to the beginning of the movement.

Legible Movement Formalism

What is the inference function that the observer has to perform?

Legible Movement Formalism

$$\mathcal{I}_L(\xi_{S\to Q}) = \hat{G} = \arg\max_{G\in\mathcal{G}} P(G|\xi_{S\to Q})$$

The guessed goal is the one that has the highest probability given the trajectory “snippet”.

Legible Movement Formalism

$$\mathcal{I}_L(\xi_{S\to Q}) = \hat{G} = \arg\max_{G\in\mathcal{G}} P(G|\xi_{S\to Q})$$

How do we compute this probability?

Legible Movement Formalism

Legible Movement Formalism

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Data

Prior

Posterior

Normalizing constant

Legible Movement Formalism

$$P(G|\xi_{S\to Q}) = \frac{P(\xi_{S\to Q}|G)P(G)}{P(\xi_{S\to Q})}$$

Data

Prior

Posterior

Normalizing constant

Legible Movement Formalism

All possible trajectories that pass through Q to G

$$P(\xi_{S\to G}|G) = \frac{\int_{\xi_{Q\to G}} P(\xi_{S\to Q\to G})}{\int_{\xi_{S\to G}} P(\xi_{S\to G})}$$

All possible trajectories from the start to the goal.

Legible Movement Formalism

Move this part of the trajectory out because it's already given (we know it!).

$$P(\xi_{S\to G}|G) = \frac{P(\xi_{S\to Q}) \int_{\xi_{Q\to G}} P(\xi_{Q\to G})}{\int_{\xi_{S\to G}} P(\xi_{S\to G})}$$

Legible Movement Formalism

The robot needs an idea of how probable a movement is in the eye of the human interactor.

$$P(\xi_{S\to G}|G) \propto \frac{exp(-C(\xi_{S\to Q}))\int_{\xi_{Q\to G}} exp(-C(\xi_{Q\to G}))}{\int_{\xi_{S\to G}} exp(-C(\xi_{S\to G}))}$$

Same cost function as earlier.

Legible Movement Formalism

The robot now knows:

1. Which the most probable action is given the goal.

$$P(G|\xi_{S\to Q}) = \frac{P(\xi_{S\to Q}|G)P(G)}{P(\xi_{S\to Q})}$$

Legible Movement Formalism

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1. Which the most probable action is given the goal.

$$P(G|\xi_{S\to Q}) = \frac{P(\xi_{S\to Q}|G)P(G)}{P(\xi_{S\to Q})}$$

2. How probable that action is for the observer.

$$P(\xi_{S\to G}|G) \propto \frac{exp(-C(\xi_{S\to Q}))\int_{\xi_{Q\to G}} exp(-C(\xi_{Q\to G}))}{\int_{\xi_{S\to G}} exp(-C(\xi_{S\to G}))}$$

Legible Movement Formalism

We now need to assess how legible a movement is and perform the movement (then we’re done!)

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Make a function that favours trajectories where the goal is more obvious earlier on in the movement (it gives more weight to smaller trajectory “snippets”)

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$$f(x)$$

Legible Movement Formalism

We now need to assess how legible a movement is:

Our function that favours earlier legibility.

$$legibility(\xi) = \frac{\int P(G|\xi_{S\to Q}) f(t) dt}{\int f(t) dt}$$

Legible Movement Formalism

We now need to ~~assess how legible a movement is and~~ perform the movement (then we're done!)

We don't want crazy movements!

Legible Movement Formalism

We now need to ~~assess how legible a movement is and~~ perform the movement (then we're done!)

$$L(\xi) = legibility(\xi) - \lambda C(\xi)$$

Ensures that the legibility score doesn't get too large

What does all this imply?

Despite what intuition might say:

Legible and predictable motion can be non-complementary.

More predictable movements might be less legible and visa-versa.